

## **Rent Gradient of a College Town**

George Langelett\*, Kuo-Liang Chang\*\*, Ekaterina Koromyslova<sup>^</sup>,  
Charles Appiah<sup>^</sup>, Jessie Duncan-Williams<sup>^</sup>, Michael Foote<sup>^</sup>,  
Daniel Johns<sup>^</sup>, Liming Lai<sup>^</sup>, Zelie-Sandra Munzimi<sup>^</sup>,  
Daniel Rezac<sup>^</sup>, Chi Wang<sup>^</sup>, and Liangchuan Zhou<sup>^</sup>

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\*George Langelett ([george.langelett@sdstate.edu](mailto:george.langelett@sdstate.edu)) is an Associate Professor of Economics.

\*\*Kuo-Liang ([kuo-liang.change@sdstate.edu](mailto:kuo-liang.change@sdstate.edu)) is an Assistant Professor of Economics.

<sup>^</sup> Denotes graduate students in Econ 576 Marketing Research course at SDSU.

Lead authors are located in the Department of Economics at South Dakota State University, Box 504 Scobey Hall, Brookings, SD 57007-0895 (605-688-4141).

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### **Abstract**

Metropolitan areas exhibit a rent gradient based on the distance from the Central Business District (CBD). By contrast to metropolitan models, Brookings, SD, is a traditional college town and home to South Dakota State University. The city also has a vibrant business district located on Main Avenue. Based on model construction from previous research, data gathered from rental units in Brookings were analyzed and compared. According to the empirical results, a rent gradient exists around campus rather than the traditional CBD.

## **I. Introduction**

Numerous studies have researched the conventional household location model, where the change in rents is based on distance from the city center. It includes transportation costs from outlying districts to the center and is often estimated using a monocentric model, where rents decline at a decreasing rate with distance from the Central Business District (CBD). Thus, the expected sign for the first derivative with respect to distance is negative and the second derivative, positive. The monocentric model of urban spatial development was developed by Alonso (1964), Mills (1972), and Muth (1969). In this theoretical model, all employment is concentrated at the Central Business District (CBD) and location choice is modeled solely on the basis of access to the employment center. A key implication of the monocentric model is that the rental value per unit of housing service declines by distance from the CBD. The model explains the location decision of residents based on the choice between closeness to the CBD, meaning less travel time and expenses, and a suburban/rural locality, meaning greater commuting costs, but lower housing prices.

Our study differs from previous work as we apply the rent gradient model to a college town. Brookings, SD, is a traditional college town and home to South Dakota State University. The city also has a vibrant business district located on Main Avenue. Although South Dakota is well known for agriculture, Brookings is home to a notable number of manufacturing companies, including Falcon Plastics, Daktronics Inc., Larson Storm Doors, and Rainbow Play Systems. 3M Inc. has a medical-tape factory and Bel Brand Cheese is building a cheese plant in Brookings. The purpose of this study is to test the possibility of the existence of a poly-centric rent gradient: one gradient around the downtown, the traditional center for monocentric land-use models, and one around the university, the logical center for a college town.

## II. Literature Review

The Urban rent gradient has been studied extensively for large metropolitan communities. For example, Filatova et al, (2009) presented an agent-based model where all individuals in the city commute to work in the Central Business District (CBD). Commuting costs are assumed to be linear by distance, and an individual must reach the same level of utility regardless of location in the city. Thus, transaction prices are equal for all cells at an equal distance from the CBD. Locations further from the CBD imply higher commuting costs; in order for individuals to be willing to move there, housing rents must fall enough to compensate the individual for the greater commuting cost. This is the concept of a bid-rent function; it shows how much an individual is willing to pay for housing (per square foot) at a particular location. The bid-rent function declines with distance from the CBD to compensate for increased commuting cost. The authors estimated a functional representation of the rent gradient through a linear regression analysis. They concluded that at longer distances from the CBD, the more commuting-tolerant buyers bid higher than the people with strong preferences for proximity to the CBD.

Likewise, in “Transportation Cost and Rent Gradient”, Coulson and Engle(1987) presented another reason the price of housing is negatively correlated with distance from the downtown area. Increases in transport costs will raise the price of centrally located houses. The empirical results from Coulson’s study showed this to be the case, but in an unexpected way. While changes in the nongasoline transportation costs appear to have been capitalized into housing prices, changes over time and gasoline costs have been overcapitalized.

In another empirical paper, Plina Veksler (2002) tested the possibility that two cities, Moscow in Russia and Houston in Texas, may have almost identical apartment rent gradients. A hedonic regression was performed using the natural logarithm of rent as the dependent variable. The explanatory variables were the log of the distance from the city center, the log

of the area of the apartment, and number of bedrooms. The study showed that even though the two cities could not have been more different in historical development, Moscow's rent gradient was similar to that of Houston.

Instead of testing two separate cities, Wilson and Frew(2007) studied changes in the urban rent gradient at two points in time. They examined changes in the apartment-rent gradient of Portland, Oregon, a mono-centric city, before and after the technology boom, from 1992 to 2002. The empirical model was a hedonic rent equation augmented with distance controls. Three distances were measured for each apartment observation in the sample: the commuting distance to the freeway onramp; the distance to the nearest freeway intersection; and the distance to the city center. The results indicated that population growth increased apartment rents across the metropolitan area. The largest increases occurred adjacent to the city center and along the beltway.

As opposed to the Alonso-Mills-Muth rent-gradient model, a number of papers have studied caveats to the traditional model. For example, Bo Söderberg and Christian Janssen (2001) studied distance effects by including distance-related explanatory variables in hedonic models. Data were collected for apartment properties in Stockholm, Sweden. The authors estimated that the rent gradient did not exist where the market would place the city center (as implied by the observations). This anomaly was caused by the city's strict rent control ordinances. However, across the greater region, they were able to detect a significant negative price gradient.

Colwell and Munneke (1997) suggested that a nonlinear modeling may be preferable because the existence of nonlinear land prices impacts the rate of price decline by distance from the urban center. The authors hypothesized that concave parcel prices result in an over-estimation of the rate of price decline by distance, because parcel sizes grow larger with distance. The concavity of the land price function is caused by larger parcels costing less per

square-foot away from the CBD. The hypothesis was empirically tested using residential, commercial, and industrial land sales. The location of all sales and other relevant positions in space were mapped using the rectangular survey system. The empirical results confirmed that land prices were concave with regard to parcel size. Modeling for concavity greatly reduced, but did not eliminate the decline of land prices with distance.

Dubin and Sung (1987) demonstrated several potential downward biases resulting from the monocentric assumption. Four factors that affect rent prices, other than housing characteristics and distance from the CBD, are: 1) non-CBD employment centers, 2) universities, cultural centers, and parks, 3) transportation routes, and 4) racial composition of the neighborhood. Thus, the CBD is not the only location to which access is important. The study found that employment and amenity centers, both suburban and in the CBD, were correlated with the expected gradients on housing prices; but, their influences were limited, and they only affected housing prices within a 1 to 1.5 mile radius of each center. Finally, the monocentric rent gradient resulting from the CBD was not found in any of the estimations.

An alternative to the Alonso-Mills-Muth rent gradient model was hypothesized by Richardson (1977). He proposed a model in which the rent distance function may be upward sloping instead of downward-sloping, even if all individuals are identical and land differs only in its distance from the city center. The critical assumption of Richardson's model is that average neighborhood plot size directly enters each resident's utility function. He asserted that the bid-rent model of Alonso-Mills-Muth is an unnecessarily strict assumption of urban land rent because it ignores the possibility of positive externalities resulting from lower residential density with distance from the CBD.

Helen Tauchen (1981) demonstrated, through a theoretical model, that the possibility of positive rent gradients is not a priori inconsistent if the concept of "equilibrium" is introduced into Richardson's model. Because finding equilibrium was not part of

Richardson's original work, the purpose of Tauchen's model was to show through a Pareto-optimal allocation, that the slope of the rent gradient is determined by the concavity of the utility function and also by the substitutability of individual and average neighborhood plots sizes.

By contrast, Grieson and Murray (1981) disagreed with the Richardson model. They analytically rebutted Richardson's assumption that a positive residential rent gradient can result from positive externalities of lowered population densities. They presented their rebuttal using a variety of different cases, including: optimal zoning, suboptimal zoning including excessive restrictions, and no internalization of externalities. In each case, a positive rent gradient is either unstable or impossible; therefore, the traditional negative gradient of the Alonso-Mills-Muth model becomes a more palatable conclusion.

Coulson (1991) outlined the difficulties of verifying the predictions of the monocentric land-use model. Two major quantifiable hypotheses which the standard rent gradient model generates are: (1) the price of land declines with distance from the CBD; and (2) the consumption of land per household increases with distance from the CBD. The monocentric model was tested in the hedonic framework using both linear and Box-Cox functional forms. Holding other attributes constant, the results of the empirical tests confirmed the predictions of the urban rent gradient model.

Youngsun Kwon(2002) developed an alternative to the rent-distance model called the rent-commuting-cost function in order to better explain the urban rent gradient. The rent-commuting-cost function suggests rent is a function of the total commuting costs, rather than distance from the CBD. Because of suburbanization, the rent-distance function loses validity to measure the urban rent gradient. Utilizing a series of graphics, Kwon demonstrated that the rent-commuting cost function (RCC) predicted the urban rent gradient more precisely than the rent-distance function. The advantage of the RCC lies in the fact that the RCC function is

independent of the form of the commuting cost function. When the form of the commuting cost function is S-shaped, comparing multiple income groups' rent gradients measured at specific distances from the CBD produces results that are difficult to interpret. However, if different income groups' rent gradients are measured using a RCC function, the higher income households are less price-sensitive and result in flatter rent gradients.

Leaving the mono-centric model, a number of studies applied a polycentric model to allow for the possibility of the existence of multiple rent gradients. Franklin and Waddell (2002) examined the relationship between single family residential valuation and property accessibility in a polycentric metropolitan model. The study provided evidence on how property accessibility affects housing values, while controlling for both congestion in the transportation network and differences in accessibility based on type of employment. A Hedonic regression model was used to test property valuation based on accessibility by type of employment. Commercial, educational, university, and industrial destinations were measured as separate accessibility indices using the quantity of employees in each type. University destinations were measured using the quantity of full time equivalent students enrolled. The authors found that access by activity type, commercial, educational, industrial and university accessibility, had a significant effect on property values. However, only commercial and university accessibility were positive and could explain the effects of accessibility on housing prices. The other two effects, educational and industrial accessibility, were negative, suggesting that benefits from access to these uses are outweighed by negative effects associated with proximity.

Internationally, Lichtenberg and Ding (2008) utilized a polycentric urban gradient model. They investigated, both theoretically and empirically, how economic incentives shaped urban expansion around Shanghai, China. The authors modeled the municipal officials' primary land allocation decisions with a focus on urban-spatial expansion. The value

of urban and rural land was estimated indirectly, using a polycentric urban model for the contributions of land and other factors of production to urban and agricultural output at the county level. They estimated parameters for the disaggregated urban and agriculture sector models and concluded that current economic incentives appear to be generating the equilibrium polycentric urban gradient of Shanghai.

Leaving the hedonic regression framework, Diamond and Mason(1995) examine the urban rent gradient in a utility-based framework. They embedded distance from the CBD directly into the utility function, while allowing household preferences to vary. The research demonstrated a preference-based framework for continuing empirical work in the areas of measuring housing rent gradients and applied household demand in an urban and regional context. More work is needed to uncover the extent to which the assumption of fixed identical preferences and self-selection bias complicates the true correlation between housing price and distance from the CBD.

Finally, moving away from studies that focus on the CBD, Lewis and Kapp (1994) estimate the rent gradient around two different university campuses in Utah – Brigham Young University (BYU) and Utah State University (USU). They used a monocentric model centered around each campus. After controlling for housing related structural variables, they found a significant negatively sloped gradient around both campuses. The study found that between two blocks and six blocks away from campus, housing rent declined by \$25.62 at BYU and \$36.02 at USU. By computing expected travel time, implicitly, students understand opportunity cost and valued their time between \$4.25 and \$6.50 per hour.

### **III. Background and Hypothesis**

Many of the previous studies focused on a monocentric model for rent gradient estimation. In the Alonso-Mills-Muth model, all employment is concentrated at the CBD and location choice is modeled on the basis of access to the employment center. A key implication

of the monocentric model is that the rental value per unit of housing service declines by distance from the CBD. In our paper, we allow for the possibility of either a mono- or poly-centric rent gradient to exist in a traditional college town.

Brookings is the fourth largest city in South Dakota, with a population of 22,056 according to the 2010 census. The population density is 1,853.4/sq mi. There are 8,715 housing units in the city. Major employers are Daktronics, Inc. (2,400 estimated employees, 10.9% of population), 3M (880 estimated employees, 4% of population), Larson Manufacturing (725 estimated employees, 3.3% of population), Rainbow Play Systems (500 estimated employees, 2.3% of population), Brookings Hospital (400 estimated employees, 1.8% of population), and South Dakota State University (1,650 employees, 7.5% of population and 12,725 students, 57.7% of population). So, 65.2% of the town's population are drawn toward SDSU. Either/both Downtown Brookings and the "Union" on SDSU Campus might be considered as the CBD of Brookings, depending on if one's focus is employment or education. We assumed the possibility of two separate rent gradients with vectors from both SDSU and from Downtown as our model's CBD.

Thus, although a number of studies on a rent gradient were conducted, no studies have investigated the possibility of two different rent gradients for a college town. Therefore, the goal of our paper is to find out the existence of the rent gradient in a small college town under the assumption that Brookings may have two possible separate rent gradients. Our expectation is to observe rents declining with distance from the city center, considering that Brookings is monocentric like the traditional Alonso-Mills-Muth monocentric model. That would be the first possible rent gradient. We might also consider SDSU Campus as the CBD of Brookings because fifty-seven percent (57.7%) of the population of Brookings is the South Dakota State University students and about seven percent (7.5%) of the population is university employees. That would be the second possible rent gradient.

#### IV. The empirical design of study

111 observations of rental housing were collected in the Brookings, SD. Data were obtained on price of monthly rent, street address, and number of bedrooms from the Brookings Chamber of Commerce. Using [itouchMap.com](http://itouchMap.com), the location of each observation was found. Next, the Brookings equalization office was utilized to assess the value of each property. We also drove to each of the apartment buildings to determine the number of units in each building. Finally, the distance was determined for each of our observations from our selected locations of CBD (the SDSU Student Union) and the center of downtown (Ray's Corner). This distance was measured from each observation to both the Student Union and Ray's Corner using Boulter's Coordinate Distance Calculator. The website allows the user to input coordinates of two locations and returns a distance value in miles, kilometers, or nautical miles. For this study, distance was calculated to the nearest hundredth of a mile and then converted to city blocks.

#### V. Variables for the Model

Dependent variable: ***Rent Per Bedroom***: our observed rental units were a mixture of apartment buildings and houses. Houses/boarded houses had from one to seven bedrooms; some apartment buildings had up to 200 bedrooms. So, to standardize rent, for each building we divided total building rent by the number of bedrooms.

Independent variables: ***Campus***: distance from the rental building to campus (city blocks); hypothesized sign is expected to be negative. This suggests, based on our theoretical model, that apartments closer to campus are more desirable, and they command a premium compared to units further away. ***Downtown***: distance from rent building to downtown (city blocks); sign is also expected to be negative. Again, this suggests people are willing to pay a premium for rental units closer to downtown. ***Bedrooms***: number of bedrooms per rental unit;

sign is expected to be negative. We expect rent per bedroom to decrease as a unit's rent is divided by more bedrooms. For example, a 1-bedroom apartment may rent for \$400.00; or \$500.00 for a 2-bedroom apartment. Dividing by two bedrooms, rent becomes \$250.00 per bedroom. *Age*: age of rent building (in years from present or 2012 minus year building was built). Sign is expected to be negative. Newer rental units command a higher rent than older buildings. *Value of the Building*: Because it is difficult to compare a 126-unit apartment complex to a single bedroom house, the value of the building was divided by the number of apartments in the complex in order to arrive at a value of the building per apartment. The sign is expected to be positive. More expensive buildings command a higher rental price than buildings assessed at a lower value. Table 1 presents the descriptive statistics for the variables used in this study.

**Table 1: Descriptive Statistics for Brookings Rent Gradient Variables**

	Minimum	Maximum	Mean	Std. Deviation
Rent	\$ 125.00	\$ 850.00	\$ 328.30	\$ 116.64
Bedrooms	.50	7.00	2.6	1.5
Value (per apartment)	\$16,101.00	\$371,200.00	\$97,420.64	\$62,541.69
Distance to Campus (blocks)	.12	34.2	11.76	7.44
Distance to Downtown	.72	33.72	10.56	7.2
Year Built	1884	2009	1950.71	40.58
N = 111				

## VI. The Empirical Model of the Rent Gradient for Brookings

Many authors applied linear and log-linear regression models (for example, Veksler, Filatova, Wilson, and Franklin) and obtained relevant empirical results. In a linear regression framework, monthly housing rent is regressed on: distance to campus, distance to downtown, and three structural variables. Two specifications of the OLS regression were run--one in levels and the other as a log-log--in order to test rent elasticities.

$$7a) \text{ Rent} = \alpha_0 + \sum_{j=1}^3 \alpha_j X_j + \beta_1 \text{Downtown} + \beta_2 \text{Campus} + \varepsilon_1$$

$$7b) \ln \text{ Rent} = \alpha_0 + \sum_{j=1}^3 \alpha_j \ln X_j + \beta_1 \ln \text{Downtown} + \beta_2 \ln \text{Campus} + \varepsilon_2$$

Where:  $\ln$  is the natural logarithm;  $X_j$  is a structural variable; and  $\varepsilon$  is an error term.

SAS software was used for regression analysis of data and for testing the significance of the variables. Coefficient of determination and a test of heteroskedasticity were used for model fitting evaluation. Finally, each variable's Variance Inflation Factor was run to test for possible multi-co-linearity problems.

## VII. Results

Levels Specification:  $\text{rent} = f(\text{campus}, \text{downtown}, \text{age}, \text{bedrooms}, \text{value})$

**Table 2-Results of Levels Regression**

Variable	Parameter Estimate	Standard Error	t Value	Pr >  t
<b>Intercept</b>	497.77	35.12		
<b>campus</b>	-4.12	1.43	-2.88	<0.01
<b>downtown</b>	-2.63	1.89	-1.39	0.16
<b>Age</b>	-1.41	0.32	-4.38	<0.01
<b>bedrooms</b>	-16.17	6.41	-2.52	0.01
<b>Value</b>	0.00	0.00	2.80	<0.01
<i>Root MSE</i>	97.11	<i>R-Square</i>	0.34	
<i>F Value</i>	10.73	<i>Pr &gt; F</i>	<.0001	

Log-Log Specification:  $lrent = f(lcampus, ldowntown, lage, lbedrooms, lvalue)$

**Table 3-Results of Log-Log Regression**

Variable	Parameter Estimate	Standard Error	t Value	Pr >  t
<b>Intercept</b>	5.55	0.43		
<b>lcampus</b>	-0.004	0.0016	-2.58	0.01
<b>ldowntown</b>	-0.005	0.0033	-1.52	0.12
<b>lage</b>	-0.14	0.03	-5.56	<0.01
<b>lbedrooms</b>	-0.13	0.04	-3.34	<0.01
<b>lValue</b>	0.08	0.03	2.56	0.01
<i>Root MSE</i>	.24	<i>R-Square</i>	0.38	
<i>F Value</i>	12.91	<i>Pr &gt; F</i>	<.0001	

In the levels specification, the three structural variables: *age*, *number of bedrooms*, and *property value* are all statistically significant at the 1% level and of the expected sign. The coefficient of zero on the *property value* variable suggests that marginal effect of adding one additional dollar to the value of a rental unit will be very small in terms of the resulting changes in rent. The statistically significant coefficient on the *Campus* variable of -4.12 suggests that a rental unit located approximately one city block from the university can expect to rent for fewer dollars than a unit located next to campus. The *Downtown* variable is insignificant, even at the 10% level. This suggests that there is no rent gradient around the downtown of Brookings, South Dakota.

For the Log-Log specification, the three structural variables are all again statistically significant at the 1% level and of the expected sign. The coefficient on the *Log-Value* variable suggests that on average, an investor in Brookings can expect around an 8% return on investment. The *Log-Campus* variable is statistically significant at the 1% level, and the coefficient of -.004 suggests a .4% reduction in rent for a rental unit located one city block away from campus than a comparable unit next to campus. The *Log-Downtown* variable is

again insignificant, even at the 10% level. The *Log-Downtown* variable suggests that a rent gradient around the downtown area does not exist. *Log-age* and *Log-Value* both have relatively large coefficients, suggesting that not only location is important in determining rental prices, but also the age and the quality of the rental property have a major impact on the rent price for which any given residential property can be rented.

### VIII. Visual Representation of the Rent Gradients in Brookings

Although tables 1, 2, and 3 provides the statistical overview of the data involved in this study, it is impossible to understand the nature and contour of the rent gradient in Brookings SD. Therefore, figure 1 and figure 2 provide a visual aid for rent gradient of in Brookings, S.D. Figure 1 and figure 2 are linear regression lines fitted to the data points for rental properties in Brookings. Figure 1 presents the rent gradient around the South Dakota State University Campus, with a value of zero on the horizontal axis representing the edge of campus. Likewise, figure 2 is the predicted rent gradient around downtown Brookings.

**Figure 1: Rent Gradient around South Dakota State University**

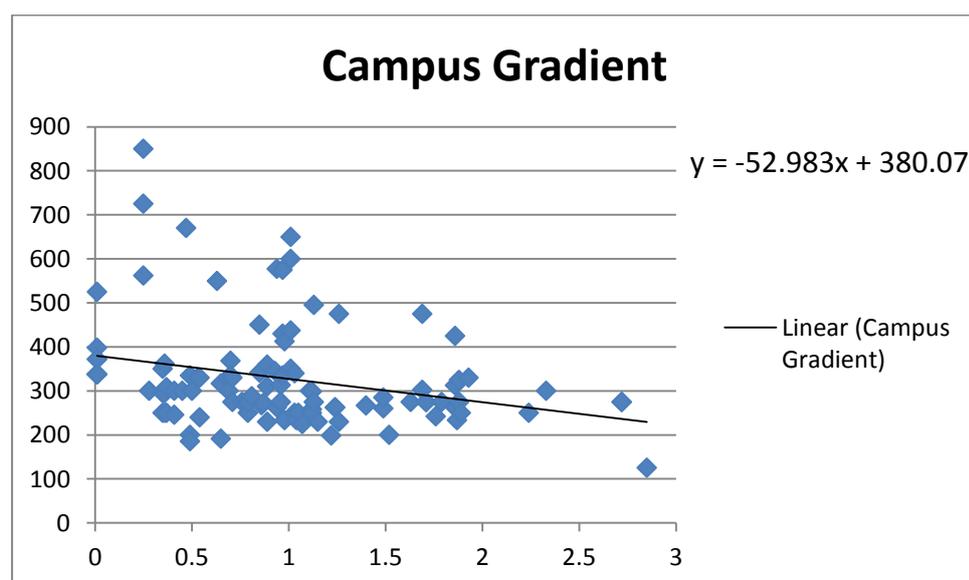


Figure 1 suggests that on average, around the edge of campus rent is \$380.07 per bedroom, and for every mile away, the price of rent falls by \$52.98; or for every city block, rent falls by \$4.42.

**Figure 2: Rent Gradient around Downtown Brookings**

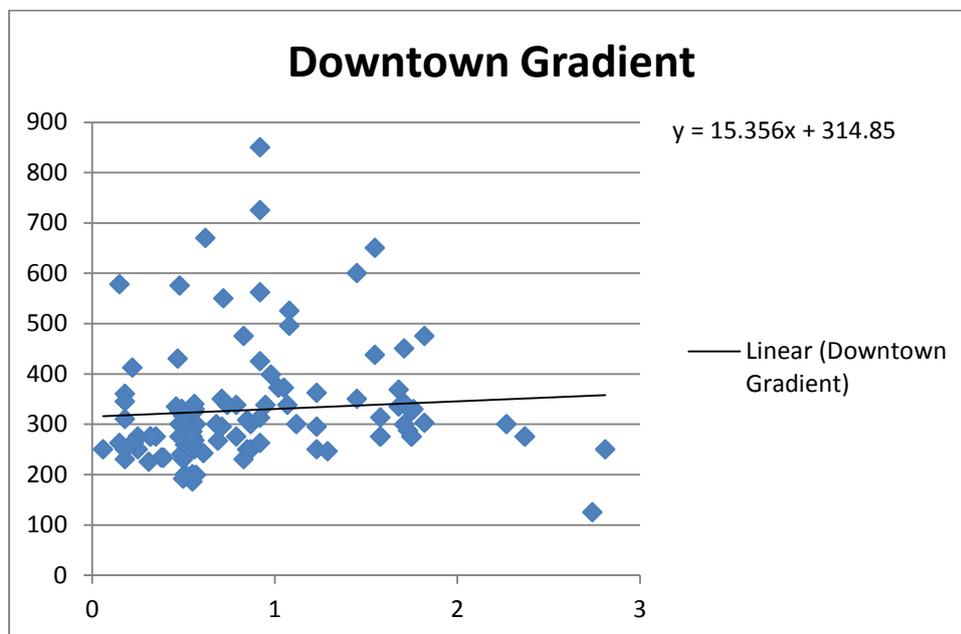


Figure 2 suggests that on average, around downtown Brookings, rent is \$314.85 per bedroom, and for every mile away, the price rises by \$15.36; or for every city block, rent rises by \$1.28.

## IX. Conclusions and Recommendations

Previous studies suggest the existence of either a monocentric or a polycentric rent gradient. We investigate this possibility in a traditional college town. Based on work by Alonso (1964), Mills (1972), and Muth (1969), we develop theory, based on the behavior of a representative resident, which could lead to the existence of either a mono- or poly-centric rent gradient in a traditional college town. Our empirical results suggest that Brookings SD has a mono, rather than polycentric rent gradient. The monocentric gradient model of rental unit pricing is centered at South Dakota State University, rather than the traditional business

district. Although the rent gradient is centered at the university, throughout the town, the age and amenities of any given building also significantly affect the rental rate for each property.

### References

- Alonso, W. (1964) *Location and Land Use*. Cambridge MA: Harvard University Press.
- Colwell, Peter F. and Henry J. Munneke. (1997). The Structure of Urban Land Prices, *Journal of Urban Economics*, 41, 321-336.
- Coulson, Edward N. (1991). Really Useful Tests of the Monocentric Model, *Land Economics*, 67(3), 299-307.
- Coulson, Edward N. and Robert F. Engle. (1987). Transportation Cost and Rent Gradient, *Journal of Urban Economics*, 21, 287-297.
- Diamond, Charles, and Gerety Mason. (1995). Flexible Form Methods for Measuring Rent Gradients, *Journal of Regional Science*, 35(2), 245-266
- Dubin, Robin A. and Chein-Hsing Sung. (1987). Spatial Variation in the Price of Housing: Rent Gradients in Non-Monocentric Cities, *Urban Studies*, 24, 193-204.
- Filatova, Tatiana, Dawn Parker, and Anne van der Veen. (2009). Agent-Based Urban Land Markets: Agent's Pricing Behavior, Land Prices and Urban Land Use Change, *Journal of Artificial Societies and Social Simulation*, 12(13).
- Franklin, Joel P. and Paul Waddell. (2002). Hedonic Regression of Home Prices in King County, Washington using Activity-Specific Accessibility Measures, *University of Washington*
- Griseon, Ronald E. and Murray, Michael P. (1981). On the possibility and Optimality of Positive Rent Gradient, *Journal of Urban Economics*, 9, 275-285
- Kwon, YoungSun. (2002). Rent-Commuting Cost Function versus Rent-Distance Function, *Journal of Regional Science*, 42(4), 773-791
- Lewis, Cris, and Kapp, Tim J. (1994). The Rent-Distance Tradeoff for Student Housing: An Empirical Analysis. *Journal of Regional Analysis and Policy*, 24(1).
- Lichtenberg, Erik and Chengri Ding. (2008). Local Officials as Land Developers: Urban Spatial Expansion in China, *University of Maryland*
- Mills, E.S. (1972). *Studies in the Structure of the Urban Economy*. Baltimore, MD: John Hopkins Press.
- Muth, Richard F. (1969). *Cities and Housing*. Chicago: University of Chicago Press.
- Richardson, Harry. (1977). On the Possibility of Positive Rent Gradients, *Journal of Urban Economics*, 4(1): 60-68.

- Söderberg, Bo and Christian Janssen. (2001). Estimating Distance Gradients for Apartment Properties, *Urban Studies*, 38(1), 61-79.
- Tauchen, Helen, (1981). The Possibility of Positive Rent Gradients Reconsidered. *Journal of Urban Economics*, 9, 165-172.
- Veksler, Plina. (2002). Rent Gradients, *Journal of Undergraduate Research*, 3(10).
- Wheaton, William C. (2002). Commuting, Ricardian Rent and Housing Appreciation in Cities with Dispersed Employment and Mixed Land-Use, *Department of Economics, Center for Real Estate, MIT Cambridge, Mass 02139*
- Wilson, Beth and James Frew. (2007). Apartment Rents and Locations in Portland, Oregon: 1992 – 2002, *Journal of Real Estate Research* , 29(2), 201-217